THE HARVESTING DECISION WHEN A STANDING FOREST HAS VALUE

RICHARD HARTMAN*
University of Washington

This paper analyzes the optimal harvest age of a forest if the forest provides a flow of valuable services while standing in addition to the value of the timber when it is harvested. A basic conclusion is that the presence of recreational or other services provided by a standing forest may well have an important impact on when or whether to harvest.

The determination of the optimal harvest age for a growing forest has received a great deal of attention over the decades and is, in fact, one of the major examples used in those problems of capital theory dealing with optimal duration and rotation.[1] Moreover, it is still an area of lively interest,[2] undoubtedly because of the controversy among analysts and the disparity between theory and practices.

In the formal models used to analyze these problems, the only economic value of a forest is the lumber it produces. Many of the more recent discussions have noted in passing that a standing forest may provide flood control, recreational, or other services, but these services are excluded from formal consideration. In this note, a simple model is proposed which incorporates a flow of value from a standing forest into the more traditional models. The model is then used to examine the question of when a forest should be harvested, if at all.

We shall be considering a forest growing on a given plot of land. All trees must be harvested simultaneously; hence, the forest can be considered as a unit. It will be assumed that lumber prices remain constant over time and that the forest is small relative to the aggregate supply.

Let $G(t)$ be the stumpage value of the lumber in a forest of age t. The function $G(t)$ will have the general shape of the growth curve. A typical example is illustrated in Figure 1. The slope of the function is initially positive and increasing, the slope later begins to decrease, then becomes negative, and finally the function levels off as the forest reaches a mature steady state. In the usual analysis the only range of economic interest is where the slope of this curve is positive; however, this will no longer be true if the standing forest also provides a flow of services.

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1. See Bentley and Teeguarden (1956), Gaffney (1960), and Hirshleifer (1970, pp. 82-90) for a discussion of some of the models used to approach the problem.

2. As an example, consider the recent symposium, *The Economics of Sustained Yield Forestry*, held at the University of Washington, Seattle, Washington, November 23, 1974. This note is a reaction to the papers presented by Hirshleifer and Samuelson at that symposium.

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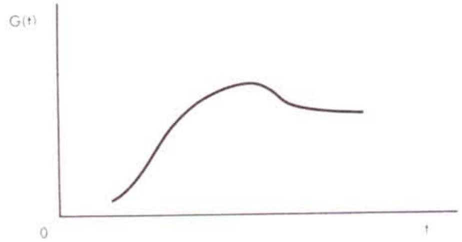
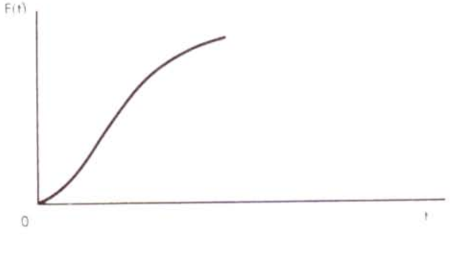
FIGURE 1



The value of the recreational and other services flowing from a standing forest of age t will be denoted by $F(t)$. To simplify the exposition, we shall simply call these services recreational services. It seems highly plausible to posit that $F(t)$ has the general shape shown in Figure 2; initially a positive and increasing slope followed by a decreasing but still positive slope. The first region incorporates the possible flood control value, the food value of a young forest to game animals, and the increasing recreational opportunities as the trees grow older. Eventually, as the trees age, the additional recreational value will increase at a decreasing rate. However, it seems reasonable to assume that the recreational value never decreases with age, particularly with the emphasis now being placed on the recreational value of "virgin" and old growth forests.

FIGURE 2



The analysis is considerably clearer and more intuitive if we begin by considering a model in which the planning horizon runs through only one cutting of the forest.[3] (A more realistic approach is introduced later.) In this case, the objective is to maximize the integral of the flow of discounted recreational values of the forest while it is standing plus the

3. This generalizes the "Fisherian" model in which the standing forest itself provides no valuable services. See Gaffney (1960) or Hirshleifer (1970, pp. 82-87).

discounted value of the timber when the forest is harvested. Mathematically, the problem is to choose t to maximize

$$V(t) = \int_0^t e^{-rx} F(x)dx + e^{-rt}G(t) \tag{1}$$

where r is the discount rate and t is the harvest age. Using the fundamental theorem of calculus, the first-order condition for an interior maximum is

$$V'(t) = e^{-rt}[F(t) + G'(t) - rG(t)] = 0 \tag{2}$$

which reduces to

$$F(t) + G'(t) = rG(t) \tag{3}$$

or

$$G'(t)/G(t) = r - F(t)/G(t). \tag{4}$$

The second-order condition is

$$V''(t) = -re^{-rt}[F(t) + G'(t) - rG(t)] + e^{-rt}[F'(t) + G''(t) - rG'(t)] < 0 \tag{5}$$

which, after using (2), simplifies to

$$F'(t) + G''(t) < rG'(t). \tag{6}$$

Hence, for an interior maximum $F(t) + G'(t)$ must intersect $rG(t)$ from above.

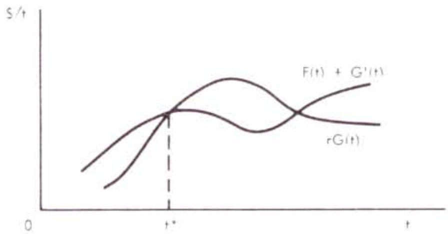
The optimality condition (3) can be interpreted easily. On the right is the interest foregone by postponing harvesting the forest for one period. On the left is the gain from postponing the harvest one period; it consists of the recreational value during the period plus the value of the timber growth over the period. Obviously, for optimality the (marginal) gain from postponing the harvest one period must equal the (marginal) loss of postponement.

In the absence of recreational value, $F(t) \equiv 0$, and (4) simply reduces to the well-known result that a forest should be harvested when its rate of growth equals the discount rate. With recreational value, however, $F(t)/G(t) > 0$, and the forest should be harvested when the rate of growth is less than the discount rate. Naturally, this is achieved by delaying the harvest. $F(t)/G(t)$ is the ratio of the recreational value per time period of the standing forest to the stock value of the harvested timber. If this ratio is greater than the discount rate, as seems likely for many forest areas, the right hand side of (4) is negative. Thus, it may well be that the optimal age for harvesting a forest involves a negative rate of growth if, in fact, it is optimal to harvest the forest at all.

The first-order condition, (3), clearly does not necessarily imply that $G'(t) > 0$ at the optimum. Moreover, the second-order condition will be satisfied for $G'(t)$ negative provided $G''(t)$ is "negative enough"; hence, the optimum may occur on a falling portion of the growth curve. Finally, if the $F(t)$ function is "large enough," there may be no solution to (3); in this case, $V'(t) > 0$ for all t, and the forest should never be harvested.

Some of these possibilities are shown graphically in Figures 3-5. In Figure 3, either intersection of the two curves satisfies the first-order condition, (3), but only at age t^* is the second-order condition also satisfied. In this case, t^* lies on the rising part of the $G(t)$ curve as it would if there were no recreational value.

FIGURE 3



In Figure 4, the recreational values play a relatively greater role than in Figure 3, and it is now optimal to harvest at age t^* where the growth rate of the forest is negative.

Finally, Figure 5 depicts a situation in which the recreational values are sufficiently great that it is optimal never to harvest. For the case depicted in Figure 5, it is always true that

$$F(t) + G'(t) - r\,G(t) > 0 \tag{7}$$

and hence, $V'(t) > 0$ for all t; the value of the forest increases the farther the harvest date is pushed into the future.

Up to this point, we have assumed that the planning horizon runs only through the first harvest. We now drop this assumption and consider a model with the planning horizon running through an indefinite sequence of harvests.[4]

4. This model is a generalization of the rotation problem and our solution is a generalization of the Faustmann solution. See Gaffney (1960) or Hirshleifer (1970, pp. 88-90).

FIGURE 4

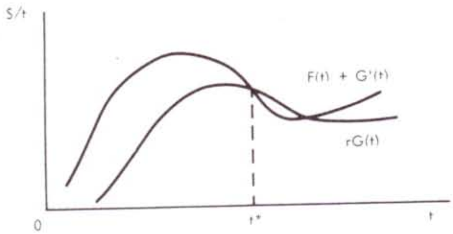
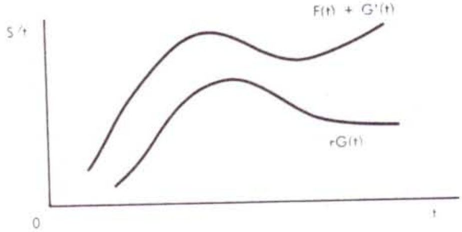


FIGURE 5



We assume that the recreational and timber stumpage valuations depend on the age of the forest and not on calendar time. To simplify the analysis, planting costs and other outlays will be ignored. (Harvesting costs are implicitly being considered since $G(t)$ refers to stumpage values.) Under these conditions, the optimal age of harvest will be the same for each growth of timber, and this age or rotation period will be denoted by t.

The objective now is to maximize

$$U(t) = G(t)[e^{-rt} + e^{-2rt} + e^{-3rt} + \ldots] \tag{8}$$

$$+ \int_0^t e^{-rx} F(x)dx[1 + e^{-rt} + e^{-2rt} + \ldots]$$

M3

$$= \frac{G(t)e^{-rt} + \int_0^t e^{-rx} F(x)dx}{1 - e^{-rt}}.$$

The first-order condition for maximization of $U(t)$ is

$$0 = U'(t) = [-re^{-rt}G(t) + e^{-rt}G'(t) + e^{-rt}F(t)]/(1 - e^{-rt}) \tag{9}$$

$$-[G(t)e^{-rt} + \int_0^t e^{-rx} F(x)dx]\,(re^{-rt})/(1 - e^{-rt})^2.$$

This simplifies to

$$(10) \qquad \frac{G'(t)}{G(t)} = r\left\{ \frac{1}{1-e^{-rt}} + \frac{\int_0^t e^{-rx}F(x)dx}{G(t)(1-e^{-rt})} \right\} - \frac{F(t)}{G(t)}.$$

Except for the term in braces, (10) is the same as (4). Loosely speaking, the term in braces acts as a "correction factor" for the interest rate. Now, $1 - e^{-rt}$ lies between zero and one, and therefore, $1/(1 - e^{-rt})$ is greater than one. Moreover, $G(t)$ and $\int_0^t e^{-rx}F(x)dx$ are both positive.[5] Thus, the expression in braces is greater than one giving rise to an "effective interest rate" (the interest rate multiplied by the "correction factor") which is greater than the interest rate appearing in (4). A review of the earlier discussion shows that this has the effect of reducing the optimal harvest age relative to the model with a one-harvest horizon. As in the simpler model, the first-order condition does not necessarily imply $G'(t) > 0$ at the optimum, and it is quite possible that it is optimal never to harvest.

The basic conclusion of this analysis is that the presence of recreational or other services provided by a standing forest may well have a very important impact on when or whether a forest should be harvested. Those models which consider only the timber value of a forest are likely to provide incorrect information in the many cases where a standing forest provides a significant flow of valuable services.

The two models considered in this note are particularly simple. In any realistic model, regeneration costs and the costs of making recreational services accessible to people would have to be explicitly considered. Moreover, care would have to be given to choosing the particular plot of land which we have called a forest. For many plots of forest land which could reasonably be taken as units for making cutting decisions, what happens on one plot will clearly affect the value of a standing forest on other plots. For example, clear-cutting part of a forested valley may have a considerable impact on the recreational value of (the scenery from) a nearby ridge. In such situations, a more complicated model, taking account of this interdependence, is needed.

5. For $G(t)$ to be positive, stumpage values must be positive or, equivalently, harvesting costs must be covered. At the optimal harvest age, this must be true, for if it were not, the forest would never be harvested.

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