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Optimal forest stock and harvest with valuing non-timber benefits: a case of US coniferous forests

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Abstract

How much to cut and to remain, as well as when to cut is an important decision-making issue in forest management. Unlike forest age, forest stock and harvest levels are applicable to both plantations and natural forests. This paper investigates the optimal forest stock and harvest with the consideration of both timber and non-timber benefits. The impacts of the discount rate, silvicultural cost, marginal timber benefit, and marginal non-timber benefit on the optimal forest stock and harvest are also examined. The results indicate that forest stock should be thickened when non-timber benefits are valued in addition to timber. The optimal steady state stock increases with a decrease in the discount rate or an increase in marginal non-timber benefit. However, the impacts of the discount rate, marginal timber benefit, and marginal non-timber benefit on the optimal steady state harvest are ambiguous. In addition, a decrease in the discount rate has the same effect on the optimal steady state stock and harvest as an increase in the ratio of marginal non-timber benefit to marginal timber benefit. These theoretical results are illustrated through an empirical example of the US coniferous forests. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

Many studies have been undertaken to determine the optimal forest rotation length under different scenarios since the advent of the Faustmann formula. Some were focused on the optimal rotation age with the consideration of only timber value (Hyde, 1980; Chang, 1983; McConnell et al., 1983; Newman et al., 1985). Others searched for the optimal rotation age with the inclusion of both timber and non-timber benefits (Calish et al., 1978; Hartman, 1976; van Kooten et al., 1995). These studies have provided important guidelines on when to cut trees in the even-aged plantations. However, their applications in uneven-aged, or natural forests, are limited because age is no longer an appropriate variable under such circumstances. Also, in formulating a forest manage-

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ment plan/policy, particularly in the management of a large-scale (such as a regional or national scale) forest resources, it may be more relevant to determine how much timber should be harvested and what level of the forest stock should be maintained than to know when trees should be cut. This will become increasingly important if non-timber benefits that largely depend on forest stock are valued, and if sustainability needs to be addressed in forest resource management and utilization. Moreover, to analyze how forest stock and harvest respond to changes in the discount rate, timber price, non-timber benefit, and silvicultural cost would also be helpful in better understanding some of the emerging issues in forest management such as deforestation, and sustainable and multiple-use management. Finally, the optimal steady state harvest level, unlike the optimal rotation age, can directly provide information on the long-run potential timber supply from a forest.

To this end, this article investigates the optimal forest stock and harvest with and without addition of non-timber benefits to timber value, and the impact of some economic and financial factors (discount rate, timber prices, non-timber benefits, and silvicultural costs) on the optimal stock and harvest. A theoretical optimal control model of forest management will be formulated. The model will then be solved to find the optimal steady state forest stock and harvest. The impact of the economic and financial factors on the optimal steady state forest stock and harvest will also be analyzed, followed by discussion on policy implications. Finally, the western and eastern US coniferous forests will be used as an empirical example to illustrate our theoretical results.

2. Methodology and theoretic approach

Forest production is a joint production process, in which inputs are transferred into multi-outputs. The outputs derived from forests may consist of timber and non-timber benefits. Assume that the timber benefit is a function of the amount of the timber harvested (h), denoted by $U(h)$, and that the non-timber benefits depend on the level of the forest stock (x), denoted by $V(x)$. It is also assumed that only two inputs are involved in the forest production. One is a composite input, silvicultural effort (E) including all inputs except land needed to generate/regenerate and manage the forest for both the timber and non-timber benefits. The other is forestland (L).

To develop a model which can be applied to different sizes of forestland and without losing the generality, this analysis is based on a unit area of forest production, i.e. $L=1$. It may be rational to assume that forest production/management is to maximize the present value of net timber and non-timber benefits over an infinite time horizon subject to the constraints of the forest stock and silvicultural efforts. Such a model for a unit area forestland can be formulated as follows:

$$\mathrm{Max}_{h(t),E(t)}\int_0^{\infty}\{U[h(t)]+V[x(t)] -wE(t)-r\}e^{-\delta t}dt \tag{1}$$

subject to:

$$\frac{\mathrm{d}x}{\mathrm{d}t}=g[x(t),E(t)]-h(t) \tag{2}$$

$$x(0)=x_0 \tag{3}$$

$$x(t)\geq 0 \tag{4}$$

Here $x(t)$ is the forest stock level at time t, $h(t)$ is the timber harvest level at time t, $E(t)$ is the silvicultural effort at time t, w is the per unit cost of the silvicultural effort, r is the rent of the forestland, δ is the continuous discount rate and x_0 is the initial forest stock level. $g(\cdot)$ is the growth function of forest stock, which is assumed to be continuous, twice differentiable, and strictly quasiconcave. These assumptions on the forest growth function seem reasonable because previous studies have shown that forest growth per unit area is generally a logistic or quadratic function of its stock (Smith, 1962; Binkley and Dykstra, 1987). For a unique interior optimum to exist, $U(h)$ and $V(x)$ are assumed to be continuous, twice differentiable, increasing, and quasi-concave in h and x, respectively. Some studies indicated that the non-timber benefit function $V(x)$ might not be increasing or concave (Calish et al., 1978; Swallow et al., 1990). The violation of these two assumptions on $V(x)$ may lead to multiple local optima, from which a global optimum can be identified, but will not affect the first-order necessary conditions for an optimum (local or global). Since this study focuses on the optimal solution and the impact of some economic and financial factors on the optimal steady state solution, the violation of the increasingness and quasiconcavity of $V(x)$ will not change the main results derived here.

Eq. (1) is the objective function that maximizes the net timber and non-timber benefits over an infinite time period. Eq. (2) is the forest stock constraint, which implies that the change rate of the forest stock is the difference between net natural growth and harvest. Eq. (3) is the initial stock constraint. Inequality Eq. (4) is the non-negativity constraint for the forest stock. Eqs. (1)–(3) and inequality Eq. (4) constitute an optimal control problem with one state variable $x(t)$ and two control variables, $h(t)$ and $E(t)$. Its corresponding present-value Hamiltonian expression is:

$$\begin{aligned} &\mathrm{H}[x(t),h(t),E(t);\lambda(t)] \\ &\quad = \{U[h(t)] + V[x(t)] - wE(t) - r\}e^{-\delta t} \\ &\qquad + \lambda(t)\{g[x(t),E(t)] - h(t)\} \end{aligned} \tag{5}$$

Here $\lambda(t)$ is the costate variable, indicating the present-value shadow price of the forest stock (x).

The current-value Hamiltonian expression can be written as:

$$\begin{aligned} &\tilde{\mathrm{H}}[x(t),h(t),E(t);\mu(t)] \\ &\quad = U[h(t)] + V[x(t)] - wE(t) - r \\ &\qquad + \mu(t)\{g[x(t),E(t)] - h(t)\} \end{aligned} \tag{6}$$

$\mu(t) = \lambda(t)e^{\delta t}$ is the current-value shadow price of the forest stock (x). The first-order optimality conditions of this problem are given by:

$$\frac{\partial \tilde{\mathrm{H}}}{\partial h} = U_h - \mu = 0 \tag{7}$$

$$\frac{\partial \tilde{\mathrm{H}}}{\partial E} = -w + \mu g_E = 0 \tag{8}$$

$$\frac{\mathrm{d}\mu}{\mathrm{d}t} - \delta\mu = -\frac{\partial \tilde{\mathrm{H}}}{\partial x} = -V_x - \mu g_x \tag{9}$$

Reorganizing Eqs. (7) and (8), we have:

$$U_h = \mu \tag{10}$$

$$w = \mu g_E \tag{11}$$

Eq. (10) suggests that at the optimal harvest level the marginal timber benefit must be equal to the current-value shadow price of the forest stock. Eq. (11) implies that the demand for factor E should be determined by equalizing the price of the factor to the current value of its marginal product.

Eqs. (2), (3), (9)–(11) along with inequalities Eq. (4) constitute a simultaneous equation system. Given functions $U(h)$, $V(x)$, and $g(\cdot)$ and the values of w, δ, and x_0, the optimal paths (traparetrics) of the controls, $h(t)$ and $E(t)$, and state $x(t)$ can be solved analytically or numerically. By setting $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}E}{\mathrm{d}t} = 0$, we can solve for the optimal equilibrium, the optimal steady state solution (x^*, h^*, E^*). For the detailed procedures regarding the solution of an optimal control problem, please refer to Clark (1990), Conrad and Clark (1987). If a forest is currently not at its steady state optimum, an optimal approach path may also be identified. The asymptotic approach and the most rapid approach paths are essentially two types of trajectories from the current state (x_0) to the steady state (x^*). The asymptotic approach assumes that $x_0 \to x^*$ as $t \to \infty$. The most rapid approach, also called a 'bang-bang' control, ensures that x_0 reaches x^* as rapidly as possible. In forest management, it means that we harvest as much as possible when

$x_0 > x^*$ or we do not harvest at all when $x_0 < x^*$ to most rapidly bring the stock to its steady state level.

3. Results

3.1. Optimal steady state forest stock and harvest

From the theoretical model presented previously, the optimal equilibrium levels of the forest stock and harvest can be derived. In addition to maximizing the present value of the net timber and non-timber benefits, the solution from the above model also represents a sustainable level of the forest stock and harvest because the optimal equilibrium is a steady state solution. Therefore, the optimal steady state forest stock and harvest are efficient and sustainable. Here we discuss two scenarios representing the consideration of: (i) only the timber value and; (ii) both the timber and non-timber benefits.

Taking the time derivative of Eq. (10) and substituting it into Eq. (9) with U_h substituting for μ [Eq. (10)], we obtain:

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{U_h}{U_{hh}}(\delta - g_x) - \frac{V_x}{U_{hh}} \tag{12}$$

At the equilibrium $\frac{\mathrm{d}h}{\mathrm{d}t} = 0$. Thus,

$$U_h(\delta - g_x) - V_x = 0 \tag{13}$$

That is,

$$\delta = g_x + \frac{V_x}{U_h} \tag{14}$$

g_x can be interpreted as the marginal growth of the forest stock, i.e. the change in the forest growth for each additional unit of its stock. Eq. (14) suggests an important relationship among the discount rate, the marginal growth of the forest stock, and the ratio of the marginal non-timber benefit to the marginal timber benefit for the optimal steady state forest stock and harvest. According to the equation, the optimal steady state forest stock and harvest should be such that the discount rate is equal to the sum of the marginal growth of the forest stock and the ratio of the marginal non-timber benefit to the marginal timber benefit.

3.1.1. Consideration of timber value only

If only timber is valued, $V_x = 0$. Since $U(\cdot)$ is increasing in h, Eq. (14) becomes:

$$\delta = g_x \tag{15}$$

Eq. (15) implies that in order to maximize the timber benefit from forest harvests, in the long run (at the steady state) we should maintain the forest stock at such a level that the marginal growth of the stock equals the discount rate. For a quasiconcave growth function $g(\cdot)$, there exist the following relationships between the forest stock (x) and the maximum sustained yield (MSY) stock (x_{MSY}):

$$x^* \begin{matrix} < \\ = \\ > \end{matrix} x_{\mathrm{MSY}}, \text{ when } g_x \begin{matrix} > \\ = \\ < \end{matrix} 0 \tag{16}$$

Hence, the optimal stock may be less than, equal to, or greater than the MSY stock, depending on the structure of the growth function. The ambiguous relationship between the optimal stock and MSY stock echoes the general relationship between the economic and MSY rotation age (Binkley, 1987). However, when the discount rate approaches zero, g_x also approaches zero, suggesting that the optimal steady state stock and harvest are equal to the MSY stock and MSY, respectively. As long as the discount rate is positive, $g_x > 0$ at the steady state, indicating that with the consideration of only the timber value, the steady state optimal forest stock and harvest are smaller than the MSY stock and MSY, respectively. If the forest stock (yield) can be considered to be positively related to its age, this is parallel to the result that the optimal rotation length is shorter than the age corresponding to MSY, founded by many studies on the optimal forest rotation (Bentley and Teeguarden, 1965; Samuelson, 1976; Hyde, 1980; Chang, 1983; Clark, 1990).

3.1.2. Consideration of both timber and non-timber benefits

Rearranging Eq. (14), we have:

$$g_x = \delta - \frac{V_x}{U_h} \tag{17}$$

At the optimal equilibrium, g_x could be positive, negative, or zero. If $\delta < \frac{V_x}{U_h}$, or $\delta = 0$, then $g_x < 0$, indicating that the optimal forest stock exceeds the MSY stock, and that the corresponding optimal harvest is smaller than MSY for a quasiconcave $g(\cdot)$. If $\delta = \frac{V_x}{U_h}$, then $g_x = 0$, implying that optimal steady state forest stock and harvest are equal to the MSY stock and MSY, respectively. If $\delta > \frac{V_x}{U_h}$, $g_x > 0$. In this situation, the optimal forest stock and harvest are smaller than the MSY stock and MSY, respectively.

Since $V(x)$ and $U(h)$ are increasing in x and h, respectively, i.e. V_x and U_h are positive, based on Eq. (17) we can infer:

$$\delta > g_x \tag{18}$$

The comparison of Eq. (15) with Eq. (18) indicates that at a given discount rate the optimal steady state forest stock when both the timber and non-timber benefits are valued exceeds that when only the timber is valued. This result is true as long as the forest growth function $g(\cdot)$ is quasiconcave in stock (x). It is independent of the function forms of $U(h)$ and $V(x)$, which, particularly $V(x)$, could be difficult to quantify. Previous studies on the determination of the optimal rotation length (Hartman, 1976; van Kooten et al., 1995) showed that the rotation period with the consideration of both the timber and non-timber benefits would be longer than that with the consideration of only timber value. For a given area of forestland, a larger forest stock means an older stand as long as the forest stock is below its carrying capacity. Therefore, our result, when applied to even-aged forest plantations, is also consistent with these previous findings. Moreover, the optimal steady state harvest level when both the timber and non-timber benefits are valued is higher (lower) than that when only the timber benefit is considered if g_x valued at the optimal equilibrium is greater (less) than zero. Comparing Eq. (15) with Eq. (17) reveals that the effect of $\frac{V_x}{U_h}$ and the discount rate on the optimal forest stock and harvest is of the same magnitude, but has opposite directions. In other words, inclusion of non-timber benefits in the determination of the optimal steady state forest stock and harvest is equivalent to reducing the discount rate by $\frac{V_x}{U_h}$. Moreover, if V_x and U_h change by a same proportion, there is no impact on steady state stock. The steady state stock increases only if V_x increases more or decreases less than U_h.

3.2. Impact of discount rate, marginal timber and non-timber benefit and silvicultural cost

The impact of changes in the discount rate (δ), marginal timber and non-timber benefit, and silvicultural cost (w) on the optimal steady state forest stock, harvest, silvicultural effort can be examined through comparative statics analyses. For simplicity, let us assume that the marginal timber benefit (U_h) and the marginal non-timber benefit (V_x) are constant, equal to positive values p and ρ, respectively. For instance, p can be considered as the timber price, and ρ the non-timber benefit of per-unit forest stock.

At the steady state, $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}E}{\mathrm{d}t} = 0$. Eq. (2) after setting $\frac{\mathrm{d}x}{\mathrm{d}t} = 0$ plus Eqs. (11) and (17) constitute an implicit equations system solving for the steady state solution. Applying the Implicit Function Theorem to these equations, we derive:

$$g_x \mathrm{d}x + g_E \mathrm{d}E - \mathrm{d}h = 0 \tag{19}$$

$$\mathrm{d}w = p(g_{Ex}\mathrm{d}x + g_{EE}\mathrm{d}E) + g_E \mathrm{d}p \tag{20}$$

$$\mathrm{d}\delta - \frac{p\mathrm{d}\rho - \rho\mathrm{d}p}{p^2} = g_{xx}\mathrm{d}x + g_{xE}\mathrm{d}E \tag{21}$$

This simultaneous equation system can be written as:

$$\begin{pmatrix} g_x & g_E & -1 \\ pg_{Ex} & pg_{EE} & 0 \\ g_{xx} & g_{xE} & 0 \end{pmatrix}\begin{pmatrix} \mathrm{d}x \\ \mathrm{d}E \\ \mathrm{d}h \end{pmatrix} = \begin{pmatrix} 0 \\ \mathrm{d}w - g_E\mathrm{d}p \\ \mathrm{d}\delta - \dfrac{p\mathrm{d}\rho - \rho\mathrm{d}p}{p^2} \end{pmatrix} \tag{22}$$

3.2.1. Impact of discount rate

To analyze the impact of changes in the discount rate alone on the optimal steady state stock, harvest, and silvicultural effort, let $\mathrm{d}\rho = \mathrm{d}p = \mathrm{d}w = 0$in Eq. (22). Also, let

$$A = \begin{pmatrix} g_x & g_E & -1 \\ pg_{Ex} & pg_{EE} & 0 \\ g_{xx} & g_{xE} & 0 \end{pmatrix} \tag{23}$$

Thus,

$$|A| = (-1)\begin{vmatrix} pg_{Ex} & pg_{EE} \\ g_{xx} & g_{xE} \end{vmatrix} = (-p)\begin{vmatrix} g_{Ex} & g_{EE} \\ g_{xx} & g_{xE} \end{vmatrix} = p\begin{vmatrix} g_{xx} & g_{xE} \\ g_{Ex} & g_{EE} \end{vmatrix} \tag{24}$$

Since $g(\cdot)$ is strictly quasiconcave and $p > 0$, $|A| > 0$. Therefore, solving Eq. (22) by using Cramer's Rule we obtain:

$$\frac{\partial x^*}{\partial \delta} = \frac{pg_{EE}}{|A|} < 0 \tag{25}$$

$$\frac{\partial E^*}{\partial \delta} = -\frac{pg_{Ex}}{|A|} \tag{26}$$

$$\frac{\partial h^*}{\partial \delta} = \frac{p(g_x g_{EE} - g_E g_{Ex})}{|A|} \tag{27}$$

Eq. (25) indicates that as the discount rate increases, the optimal steady state forest stock decreases. This is because with an increase in the discount rate, in the short run more trees will be cut and there will be less incentive for regeneration, afforestation, and forest management. As a result, in the long run (at the steady state) the forest stock will become thinner. Eq. (26) implies that the direction of the change in the optimal steady state silvicultural effort with an increase in the discount rate is ambiguous, depending upon the sign of g_{Ex}. If E and x are substitutes, i.e. $g_{Ex} < 0$, then the optimal steady state silvicultural effort increase with the discount rate. When $g_{Ex} < 0$, additional silvicultural effort will hamper the growth rate. In this case, an increases in the discount rate will enhance the long-run stock. However, if E and x are complements, i.e. $g_{Ex} > 0$, then the optimal steady state silvicultural effort moves in the opposite direction of the discount rate. One of the examples for substitutes between E and x is to fertilize an under-stocked forest. When $g_{Ex} > 0$, additional silvicultural effort will enhance the growth rate. Thus, it makes sense to invest more in silvicultural efforts to boost growth as the discount rate decreases. The sign of $\frac{\partial h^*}{\partial \delta}$ is ambiguous. Therefore, an increase in the discount rate may have a positive, negative, or zero impact on the optimal steady state forest harvest, depending upon the structure of the growth function. Interestingly, though the discount rate has a negative impact on the optimal steady state stock, it could be neutral in terms of its impact on the long-run harvest. Under a special case where $g_E > 0$ and $g_x > 0$, the steady state harvest and the discount rate may change in the same direction only if E and x are substitutes; otherwise, an increase in the discount rate will result in a reduction in the long-run harvest.

3.2.2. Impact of silvicultural cost

The impact of the per-unit silvicultural effort cost (w) on the steady state forest stock, harvest, and silvicultural effort can be analyzed by letting $\mathrm{d}\delta = \mathrm{d}\rho = \mathrm{d}p = 0$ in Eq. (22) and solving it. Using the same analogy as described previously and recalling that $g(\cdot)$ is strictly quasiconcave, we obtain:

$$\frac{\partial x^*}{\partial w} = -\frac{g_{xE}}{|A|} \quad (28)$$

$$\frac{\partial E^*}{\partial w} = \frac{g_{xx}}{|A|} < 0 \quad (29)$$

$$\frac{\partial h^*}{\partial w} = \frac{g_E g_{xx} - g_x g_{xE}}{|A|} \quad (30)$$

The sign of $\frac{\partial x^*}{\partial w}$ depends on whether E and x are substitutes, complements, or independent. It is positive if E and x are substitutes, negative if E and x are complements, and equal to zero if E and x are independent. Eq. (29) indicates that the optimal steady state silvicultural effort decreases as its per-unit cost increases. According to Eq. (30), the sign of $\frac{\partial h^*}{\partial w}$ is also ambiguous, depending the sign of $(g_E \ g_{xx} - g_x \ g_{xE})$. A deterministic relationship may exist when $g_E > 0$ and $g_x > 0$. Under this situation, the long-run harvest will decline with an increase in w if E and x are complements.

3.2.3. Impact of marginal timber benefit

Let $\mathrm{d}\delta = \mathrm{d}\rho = \mathrm{d}w = 0$ in Eq. (22). Solving the simultaneous equation system and recalling that $g(\cdot)$ is strictly quasiconcave, we have:

$$\frac{\partial x^*}{\partial p} = \frac{g_E g_{xE} + \frac{\rho}{p} g_{EE}}{|A|} \quad (31)$$

$$\frac{\partial E^*}{\partial p} = -\frac{g_E g_{xx} + \frac{\rho}{p} g_{Ex}}{|A|} \quad (32)$$

$$\frac{\partial h^*}{\partial p} = \frac{g_x g_E g_{xE} - \frac{\rho}{p} g_x g_{EE} + \frac{\rho}{p} g_E g_{Ex} - g_x^2 g_{xx}}{|A|} \quad (33)$$

The signs of $\frac{\partial x^*}{\partial p}$, $\frac{\partial h^*}{\partial p}$ and $\frac{\partial E^*}{\partial p}$ are all ambiguous. Therefore, the impact of the marginal timber benefit (p) on the optimal steady state forest stock, silvicultural effort, and harvest is uncertain. A shock in timber price may not necessarily increase or decrease the stock, harvest, and investment in silviculture in the long run. The ambiguity of the long-run effects of a timber price change on stock, harvest and silvicultural efforts seems to contradict many existing models using age classes (Hyde, 1980; Chang, 1983). However, under a special case where $g_E > 0$ and $g_x > 0$, some deterministic relationships exist. When $g_E > 0$, a boost in timber price will decrease the stock and increase the silvicultural effort when E and x are substitutes. When $g_E > 0$ and $g_x > 0$, the long-run harvest will increase with a rise in timber price when E and x are complements.

3.2.4. Impact of marginal non-timber benefits

Similarly, by letting $\mathrm{d}\delta = \mathrm{d}p = \mathrm{d}w = 0$ in Eq. (22) and solving the simultaneous equation system, we have:

$$\frac{\partial x^*}{\partial \rho} = -\frac{g_{EE}}{|A|} > 0 \quad (34)$$

$$\frac{\partial E^*}{\partial \rho} = \frac{g_{Ex}}{|A|} \quad (35)$$

$$\frac{\partial h^*}{\partial \rho} = \frac{-g_x g_{EE} + g_E g_{Ex}}{|A|} \quad (36)$$

Eq. (34) indicates that the optimal steady state forest stock increases with the marginal non-timber benefit. Obviously, when non-timber benefits are valued higher, forest stock should be increased. Eq. (35) suggests that as the marginal non-timber benefit increases, the optimal steady state silvicultural effort increases when E and x and complements, and decreases when E and x are substitutes. The ambiguity of the sign of $\frac{\partial h^*}{\partial \rho}$ implies that the optimal steady state harvest may increase, decrease, and remain unchanged as the marginal non-timber benefit increases. However, under a special case where $g_E > 0$ and $g_x > 0$, the long-run harvest and the marginal non-timber benefit will move in the same direction when E and x are complements.

3.3. US coniferous forests

The theoretical model and conclusions presented above are illustrated by using the western and eastern US coniferous forests as an empirical example. In our theoretical model, the forest growth per unit area, $g(x, E)$, is defined as a function of the forest stock (x) and silvicultural effort (E). Unfortunately, such a forest growth function is rarely available. For the simplicity of the illustration, let us assume that E is fixed. Thus, we consider forest growth as a function of its stock only. Binkley and Dykstra (1987) estimated such forest growth functions for major forests in the world. Their estimated forest growth functions for both the western and eastern US coniferous forests are used for demonstration here. The growth models they estimated are:

(a) For the western US coniferous forest:

$$g(x) = 0.0625x - 0.0002803x^2 \quad (37)$$

(b) For the eastern US coniferous forest:

$$g(x) = 0.00493x^2 - 0.0001059x^3 \quad (38)$$

Eq. (37) is a logistic function, and Eq. (38) is a cubic function. Both of them are continuous, twice differentiable, and strictly quasiconcave.

Assume that the timber benefit $U(h) = ph$, and the non-timber benefits $V(x) = \rho x$. Using Eq. (2) after dropping $E(t)$ and Eqs. (4), (7) and (9), we can solve for the optimal steady state forest stock (x^*) and harvest (h^*) as follows:

(a) For the western US coniferous forest:

$$x^* = 1783.803\left(\frac{\rho}{p} - \delta + 0.0625\right) \quad (39)$$

$$h^* = 1783.803\left(\frac{\rho}{p} - \delta + 0.0625\right) \times \left[0.0625 - 0.5\left(\frac{\rho}{p} - \delta + 0.0625\right)\right] \quad (40)$$

(b) For the eastern US coniferous forest:

$$x^* = 1573.812\Bigg\{0.009862 + \Bigg[0.000097259 - 0.00127176$$

Table 1
The optimal steady state forest stock (x^*) and harvest (h^*) under different discount rates (δ) and the ratios of the marginal timber benefit to the marginal non-timber benefit (ρ/p) for the eastern US coniferous forests[a]

ρ/p	$\delta = 0\%$		$\delta = 2\%$		$\delta = 4\%$		$\delta = 6\%$		$\delta = 8\%$		$\delta = 10\%$	
	x^*	h^*	x^*	h^*	x^*	h^*	x^*	h^*	x^*	h^*	x^*	h^*
0.00	31.042	1.584	28.859	1.561	26.240	1.482	22.725	1.304	0	0	0	0
0.01	32.025	1.579	29.992	1.579	27.620	1.530	24.653	1.410	20.037	1.128	0	0
0.02	32.954	1.565	31.042	1.584	28.859	1.561	26.240	1.482	22.725	1.304	0	0
0.03	33.835	1.543	32.025	1.579	29.992	1.579	27.620	1.530	24.653	1.410	20.037	1.128
0.04	34.676	1.514	32.954	1.565	31.042	1.584	28.859	1.561	26.240	1.482	22.725	1.304
0.05	35.481	1.477	33.835	1.543	32.025	1.579	29.992	1.579	27.620	1.530	24.653	1.410
0.06	36.255	1.435	34.676	1.514	32.954	1.565	31.042	1.584	28.859	1.561	26.240	1.482
0.07	37.001	1.386	35.481	1.477	33.835	1.543	32.025	1.579	29.992	1.579	27.620	1.530
0.08	37.722	1.332	36.255	1.435	34.676	1.514	32.954	1.565	31.042	1.584	28.859	1.561
0.09	38.421	1.273	37.001	1.386	35.481	1.477	33.835	1.543	32.025	1.579	29.992	1.579
0.10	39.098	1.208	37.722	1.332	36.255	1.435	34.676	1.514	32.954	1.565	31.042	1.584
0.15	42.230	0.818	41.024	0.987	39.757	1.139	38.421	1.273	37.001	1.386	35.481	1.477
0.20	45.032	0.329	43.945	0.535	42.814	0.728	41.634	0.905	40.399	1.065	39.098	1.208
0.25	46.563	0	46.563	0	45.561	0.220	44.493	0.434	43.385	0.633	42.230	0.818
0.30	46.563	0	46.563	0	46.563	0	46.563	0	46.081	0.108	45.032	0.329
0.35	46.563	0	46.563	0	46.563	0	46.563	0	46.563	0	46.563	0

[a] *Note*: The growth model of the eastern US coniferous forests is from Binkley and Dykstra (1987). The units of x^* and h^* are m^3/acre and m^3/acre per year, respectively.

Table 2
The optimal steady state forest stock (x^*) and harvest (h^*) under different discount rates (δ) and the ratios of the marginal timber benefit to the marginal non-timber benefit (ρ/p) for the western US coniferous forests[a]

ρ/p	$\delta = 0\%$		$\delta = 2\%$		$\delta = 4\%$		$\delta = 6\%$		$\delta = 8\%$		$\delta = 10\%$	
	x^*	h^*	x^*	h^*	x^*	h^*	x^*	h^*	x^*	h^*	x^*	h^*
0.00	111.488	3.484	75.812	3.127	40.136	2.057	4.460	0.273	0	0	0	0
0.01	129.326	3.395	93.650	3.395	57.974	2.681	22.298	1.254	0	0	0	0
0.02	147.164	3.127	111.488	3.484	75.812	3.127	40.136	2.057	4.460	0.273	0	0
0.03	165.002	2.681	129.326	3.395	93.650	3.395	57.974	2.681	22.298	1.254	0	0
0.04	182.840	2.057	147.164	3.127	111.488	3.484	75.812	3.127	40.136	2.057	4.460	0.273
0.05	200.678	1.254	165.002	2.681	129.326	3.395	93.650	3.395	57.974	2.681	22.298	1.254
0.06	218.516	0.273	182.840	2.057	147.164	3.127	111.488	3.484	75.812	3.127	40.136	2.057
0.07	222.975	0	200.678	1.254	165.002	2.681	129.326	3.395	93.650	3.395	57.974	2.681
0.08	222.975	0	218.516	0.273	182.840	2.057	147.164	3.127	111.488	3.484	75.812	3.127
0.09	222.975	0	222.975	0	200.678	1.254	165.002	2.681	129.326	3.395	93.650	3.395
0.10	222.975	0	222.975	0	218.516	0.273	182.840	2.057	147.164	3.127	111.488	3.484
0.15	222.975	0	222.975	0	222.975	0	222.975	0	222.975	0	200.678	1.254
0.20	222.975	0	222.975	0	222.975	0	222.975	0	222.975	0	222.975	0

[a]*Note*: The growth model of the western US coniferous forests is from Binkley and Dykstra (1987). The units of x^* and h^* are m^3/acre and m^3/acre per year, respectively.

$$\times\left(\delta - \frac{\rho}{p}\right)\Bigg]^{0.5}\Bigg\} \quad (41)$$

$$h^* = x^{*2}(0.004931 - 0.0001059x^*) \quad (42)$$

According to Eqs. (39)–(42), the optimal steady state forest stock and harvest for the western and eastern US coniferous forests under various levels of the ρ/p ratio and the discount rate (δ) are shown in Tables 1 and 2, respectively. These two tables suggest how much to cut and to remain in the long run (at the steady state) for 1 acre of the western and eastern US conifers at a given discount rate and a specific ratio of the marginal non-timber benefit to the marginal timber benefit.

Based on the forest growth functions [Eqs. (37) and (38)], the maximum forest stock (carrying capacity) for the western and eastern coniferous forests is 222.98 and 46.56 m^3/acre, respectively. The MSY and the MSY stock are, respectively, 3.48 m^3/acre per year and 111.49 m^3/acre for the western conifers and 1.58 m^3/acre per year and 31.04 m^3/acre for the eastern conifers.

Tables 1 and 2 show that for both the western and eastern conifers if the non-timber benefits is not valued the optimal steady state stock levels are smaller than their MSY stocks, and the optimal steady state harvest levels are also smaller than the MSY. Also, without the consideration of the non-timber benefits the optimal steady state forest stock and harvest decrease as the discount rate increases. As the discount rate approaches to zero, the optimal steady state stock and harvest approach to the MSY stock and MSY, respectively. Inclusion of the non-timber benefits increases the optimal steady state stock at a specific discount rate. However, even with the consideration of the non-timber benefits the optimal steady state stock and harvest can be smaller than, equal to, or larger than the MSY stock and MSY, respectively, depending upon the sign of $\left(\frac{\rho}{p} - \delta\right)$. As long as $\rho/p = \delta$, the optimal steady state stock and harvest should be equal to the MSY stock and MSY, respectively. When $\rho/p > \delta$, the optimal steady state harvest should be smaller than the MSY, but the optimal steady state stock should be maintained at a level larger than the MSY stock. If ρ/p is high enough relative to the discount rate, no timber should be harvested, and the forest stock will reach its carrying capacity. For example, at $\delta = 4\%$ no timber should be harvested from the western US coniferous forests if $\frac{\rho}{p} \geq 0.12$ (Table 1), and from the eastern US coniferous forests if $\frac{\rho}{p} \geq 0.28$ (Table 2). When $\rho/p < \delta$, the optimal steady state stock and harvest should be smaller than the MSY and MSY stock, respectively. However, if δ is high enough relative to ρ/p, the forests should be cleared. According to Tables 1 and 2, if the non-timber benefits are not valued, both the western and eastern conifers would be cleared if the real continuous discount rate is 8% or higher. These two tables also illustrate that an increase in ρ/p is equivalent to deflating the discount rate by the same amount in terms of the impact on the optimal steady state stock and harvest. For any combinations of δ and ρ/p, as long as $\left(\frac{\rho}{p} - \delta\right)$ is fixed, their corresponding steady state forest stock and harvest are identical.

As described previously, the optimal steady state harvest is actually the long-run sustainable timber supply. If the non-timber benefits are not valued, the western coniferous forest will supply 2.06 m^3/acre per year of timber, and the eastern coniferous forest 1.48 m^3/acre per year at a 4% real discount rate in the long run. The long-run timber supply levels (equal to h^*) for the western and eastern US coniferous forests at various ρ/p ratios and discount rates can be founded in Tables 1 and 2, respectively.

The current annual timber removal on average is approximately 0.6 m^3/acre from the western coniferous forest and 1.4 m^3/acre from the eastern coniferous forest (Waddell et al., 1989). If the current state of these forests can be thought to be around their optimal steady states, the ρ/p ratio corresponding to the current timber removal level for either the eastern or western US coniferous forests is approximately equal to 0.1 at the US Forest Service discount rate of 4% (Tables 1 and 2). This implies that the non-timber benefits we valued on average are approximately 10% of the timber prices for both the western and eastern US coniferous forests. These are the average non-timber values for the entire US coniferous forests revealed from our current timber harvest behaviors. They may not represent the non-timber value of a specific track of forest. Given the current debate on over-harvest on public timberlands, these numbers may underestimate the actual non-timber values generated from these forests.

4. Conclusions

Forest management often involves making decisions on how much to cut (harvest) and remain (stock) as well as when to cut (rotation age). Using forest stock and harvest levels as decision variables is applicable to the management of both plantations and natural forests. This paper describes an approach to determining the optimal forest stock and harvest and demonstrates the applicability of this approach in empirical forest management using the example of the US coniferous forests. Our results indicate that the optimal steady state forest stock increases when non-timber benefits are added to timber benefit. The optimal steady state stock does not exceed the MSY stock when only timber is valued. But it can be smaller than, equal to, or larger than the MSY stock when both timber and non-timber benefits are valued. The impact of the inclusion of non-timber benefits on the optimal stock and harvest is the same as that of a decrease in the discount rate by the ratio of the marginal non-timber benefit to the marginal timber benefit $\left(\frac{V_x}{U_h}\right)$. Too high a discount rate $\left(\text{relative to } \frac{V_x}{U_h}\right)$ could lead to deforestation. A high value of $\frac{V_x}{U_h}$ can help conserve forest resources. An increase in the discount rate, or a decrease in the marginal non-timber benefit reduces the optimal steady state forest stock. However, the impact of the marginal timber benefit (timber price) and silvicultural cost on the optimal steady state stock is ambiguous. Moreover, none of the discount rate, the marginal timber benefit, marginal non-timber benefit, and silvicultural cost has a certain impact on the optimal steady state harvest.

These results have some important policy implications. Discount rate has a negative impact on long-run forest stock, but its impact on long-run timber harvest/supply is uncertain. Reducing the discount rate can achieve the same objective as a subsidy to non-timber production. To enhance the long-run forest stock, marginal non-timber benefit or subsidies to non-timber production must grow more or decline less than timber price. Policies affecting only timber price do not necessarily increase and decrease long-run stock and harvest.

The results from our empirical example have several potential applications and implications. First, the optimal timber harvest levels in Tables 1 and 2 can be used to project the long-run timber supply from the western and eastern US coniferous forests. Second, the current timber removal level is close to its MSY in the eastern coniferous forest, but much below its MSY in the western coniferous forest. Therefore, the maximum sustainable yield is at least not the only criterion we currently use to manage the western conifers. Third, the ratio of the marginal non-timber benefit to the marginal timber benefit corresponding to the current timber removal level in both the eastern and western coniferous forests is approximately 0.1 at a 4% discount rate. Our current behavior in managing these forests implicitly reveals that the non-timber benefits of these conifers on average are valued at approximately 10% of their timber prices (the marginal timber benefit).

Like any models, our model has its limitations, too. First, it is a static and deterministic model. This helps simplify the study, but ignores the dynamic and stochastic nature in forest management. Second, our results are focused on steady state. The steady state assumption is significant because it represents long-run equilibrium and is related to sustainability. But, in many cases, a forest may not be or is not intended to be in steady state. Usual cautions should be taken in applying this model and its results. Further studies on dynamic and stochastic modeling and optimal trajectories to the equilibrium will certainly enrich the findings of this study.

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References

Bentley, W.R., Teeguarden, D.E., 1965. Financial maturity: a theoretical review. Forest Science 11, 76–87.

Binkley, C.S., 1987. When is the optimal economic rotation longer than the rotation of maximum sustained yield? Journal of Environmental Economics and Management 14, 152–158.

Binkley, C.S., Dykstra, D.P., 1987. Timber supply. In: Kallio, M., Dykstra, D, Binkley, C. (Eds.), The Global Forest Sector: An Analytical Perspective. John Wiley & Sons, New York, pp. 508–533.

Calish, S., Fight, R.D., Teeguarden, D.E., 1978. How do non-timber values affect Douglas-fir rotations. Journal of Forestry 76, 217–221.

Chang, S.J, 1983. Rotation age, management intensity and the economic factors of production: do changes in stumpage price, interest rate, regeneration costs and forest taxation matter. Forest Science 29 (2), 267–277.

Clark, C.W, 1990. The Mathematical Bioeconomics, 2nd ed John Wiley & Sons, New York.

Conrad, J.M., Clark, C.W., 1987. Natural Resource Economics: Notes and Problems. Cambridge University Press, Cambridge, England.

Hartman, R., 1976. The harvesting decision when a standing forest has value. Economic Inquiry 16, 52–58.

Hyde, W.F., 1980. Timber Supply, Land Allocation and Economic Efficiency. Johns Hopkins Press, Baltimore.

McConnell, K.E., Daberkow, J.N., Hardie, I.W., 1983. Planning timber production with evolving prices and costs. Land Economics 59 (3), 292–299.

Newman, D.H., Gilbert, C.E., Hyde, W.F., 1985. The optimal forest rotation with evolving prices. Land Economics 61 (5), 347–353.

Samuelson, P.A., 1976. Economics of forestry in an evolving society. Economic Inquiry 14, 466–492.

Smith, D.M., 1962. The Practice of Silviculture. John Wiley & Sons, New York.

Swallow, S.K., Parks, P.J., Wear, D.N., 1990. Policy-relevant nonconvexity in the production of multiple forest benefits. Journal of Environmental Economics and Management 19, 264–280.

van Kooten, G.C., Binkley, C.S, Delcout, G., 1995. Effect of carbon taxes and subsidies on optimal forest rotation age and supply of carbon service. American Journal of Agricultural Economics 77, 365–374.

Waddell, K.L., Oswald, D.D., Powell, D.S., 1989. Forest Statistics of the United States, 1987. USDA Forest Service Pacific Northwest Research Station Resource Bulletin PNW-RB, 168.